

Off-the-Wall Higgs in the universal Randall-Sundrum model

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ABSTRACT: We outline a consistent Randall-Sundrum (RS) framework in which a fundamental 5-dimensional Higgs doublet induces electroweak symmetry breaking (EWSB). In this framework of a warped Universal Extra Dimension, the lightest Kaluza-Klein (KK) mode of the bulk Higgs is tachyonic leading to a vacuum expectation value (vev) at the TeV scale. The consistency of this picture imposes a set of constraints on the parameters in the Higgs sector. A novel feature of our scenario is the emergence of an adjustable bulk profile for the Higgs vev. We also find a tower of non-tachyonic Higgs KK modes at the weak scale. We consider an interesting implementation of this “Off-the-Wall Higgs” mechanism where the 5-dimensional curvature-scalar coupling alone generates the tachyonic mode responsible for EWSB. In this case, additional relations among the parameters of the Higgs and gravitational sectors are established. We discuss the experimental signatures of the bulk Higgs in general, and those of the “Gravity-Induced” EWSB in particular.

KEYWORDS: Compactification and String Models, Beyond Standard Model, Higgs Physics.

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1. Introduction

The Higgs mechanism in the Standard Model (SM) provides a simple and economical explanation for electroweak symmetry breaking (EWSB). In the SM, it is assumed that the Higgs potential contains a tachyonic negative mass-squared term that causes the Higgs to develop a vacuum expectation value (vev), resulting in EWSB. However, the source of this tachyonic mass, which must be of order 100 GeV, is not explained in the SM.

Another mystery regarding the Higgs sector, pointing to physics beyond the SM, is the size of the typical Higgs mass compared to the cutoff scale; this is the usual hierarchy problem. For example, if we take the 4-dimensional (4-d) reduced Planck mass $\overline{M}_{Pl} \sim 2 \times 10^{18}$ GeV to be the cutoff scale, we would naively expect quantum corrections to raise the Higgs mass to be near \overline{M}_{Pl} , which is $O(10^{16})$ too large.

An interesting explanation of the hierarchy is provided by the Randall-Sundrum (RS) model [1]. In this model, a curved 5-d (bulk) spacetime called AdS_5 is bounded by 4-d Minkowski boundaries, corresponding to the geometry of the observed universe. The curvature of the 5-d spacetime induces a sliding scale along the warped extra dimension, geometrically generating the weak scale from a large Planckian scale, at one of the 4-d boundaries. This boundary is referred to as the TeV-brane, hereafter. The geometrical red-shifting in the RS model is exponential and hence capable of explaining large hierarchies, using parameters that are typical in the model. In the original RS model, the only 5-d

field is the graviton [1, 2]. Numerous works have since extended the RS setup to include bulk fermions and gauge fields [3]. However, even though the SM can be promoted to a 5-d theory, the fundamental Higgs field responsible for EWSB has been typically kept on the TeV-brane in the RS model. The reason for this has been to avoid problems associated with extreme fine-tuning [4] in generating TeV-scale gauge boson masses which the RS model is constructed to resolve, and conflict with known experimental data [5] including the SM relationship $M_W = M_Z \cos \theta$. Thus, the Higgs is treated as a 4-d field in the RS geometry. (There have also been attempts to build RS models without fundamental Higgs bosons [6] where the role of the Higgs doublet as a source of Goldstone bosons is played by the fifth components of the gauge fields themselves.)

In this work, we study the requirements for successful EWSB, using a *fundamental* 5-d Higgs doublet in the RS background. We show that by appropriate choices of the Higgs sector parameters in the bulk and on the branes, one can generate a single tachyonic Kaluza-Klein (KK) mode of the Higgs field in the low energy 4-d theory. This tachyonic mode is identified as the SM Higgs field. Given a suitable quartic bulk term for the Higgs, the tachyonic mode will lead to the usual 4-d Higgs mechanism and endow the electroweak gauge bosons with mass. The quartic terms will reside on the 4-d boundaries, since they are higher dimension operators in 5-d and are expected to be small in an effective theory description. Note that these terms are not necessary for the gauge invariance of the 5-d Higgs theory and thus we set them to zero at tree level in the bulk. A *novel* feature of this mechanism is that the Higgs vev now has a profile that extends into the bulk and is no longer a constant. This provides for new model-building possibilities that we will briefly discuss in our presentation. A typical signature of our scenario is the emergence of a tower of Higgs KK modes whose detection at future colliders we will consider in this work. Since all of the SM fields are now in the bulk, this scenario is an example of a *warped* Universal Extra Dimension[7].

The “Off-the-Wall Higgs” mechanism outlined above, like its 4-d SM counterpart, does not explain the origin of the mass parameters of the Higgs potential. It would be interesting if the RS geometry itself could provide the necessary mass scales of the 5-d Higgs sector, leading to novel connections between the gravity and the Higgs sector parameters. We show that a modified version of the RS gravity sector does indeed provide such a mass scale that can result in a successful realization of the Off-the-Wall Higgs mechanism. The central observation is that the most general bulk action in the RS model should include a term $\xi R\Phi^\dagger\Phi$, coupling the Ricci curvature scalar and the Higgs [8, 9]. Because there is a constant curvature in AdS_5 as well as δ -function terms on both branes, this coupling acts as a mass term for the Higgs and, with the appropriate choice of parameters, can lead to EWSB. This picture provides a link between the seemingly unrelated Higgs and gravitational parameters and eliminates the need for *ad hoc* masses in the Higgs potential.

To realize this idea, we consider the most general 5-d action for a scalar coupled to gravity, consistent with gauge symmetry and general coordinate invariance. In addition to the standard kinetic and coupling terms, this action then includes the curvature-scalar coupling, along with a string motivated[10] higher curvature Gauss-Bonnet term (GBT) [11] and brane localized kinetic terms (BKT’s) [12] for the graviton [13], as well as the afore-

mentioned boundary BKT's for the Higgs quartic coupling. In principle one could directly include *ad hoc* tachyonic mass terms for the Higgs. However, we find that, under the assumption that these terms are small, there are phenomenologically acceptable regions of parameter space where EWSB is solely driven by the gravity sector. It is interesting to note that in the favored region typical values of ξ are close to the conformal value $\xi = -3/16$.

This connection between gravity and the Higgs sectors leads to experimentally observable signals that could point to this picture as the correct mechanism of EWSB. For example, it is well-known that curvature-Higgs coupling on the TeV-brane leads to radion-Higgs mixing [8]. The same effect exists with our bulk Higgs. We discuss how measuring the Higgs-radion mixing in conjunction with other collider measurements in the Higgs and gravity sectors can test the gravity-induced EWSB scenario and establish its parameters.

In the next section, we will describe the ingredients for achieving consistent bulk Higgs mediated EWSB in the RS geometry while section 3 discusses the details of the generation of masses for the SM electroweak gauge bosons. Section 4 focuses on the gravity-induced realization of EWSB with a bulk Higgs and the new constraints on the Higgs and gravity parameters that need to be satisfied in a successful scenario. Section 5 is devoted to a discussion of experimental tests and the novel phenomenological aspects of our scenario, such as the KK Higgs physics at colliders. Here, we also outline the measurements that will result in the establishment of gravity-induced EWSB in the RS model. In section 6, we present our conclusions.

2. Bulk Higgs EWSB in RS: the general framework

In this section we will describe the overall framework for our model and the general mechanism of bulk Higgs EWSB in the RS scenario. In particular we will demonstrate how the SM gauge boson masses are generated and the appearance of the Higgs KK spectrum. We perform our analysis using the standard RS geometry [1]: two branes are located at the fixed points of an S^1/Z_2 orbifold; between the branes, which are separated by a distance πr_c , is a slice of AdS_5 and the metric is given by

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \tag{2.1}$$

Here, $\sigma = k|y|$ with k being the so-called curvature parameter whose value is of order the 5-d Planck/fundamental scale, M , and y is the co-ordinate of the fifth dimension. In order to be as straightforward as possible and to present the essential features of the model we will delay any discussion of the localization of the SM fermions until a later section and for now only assume that the SM gauge and Higgs fields are in the bulk. With this caveat our action is given by

$$S = S_{\text{Higgs}} + S_{\text{gauge}}, \tag{2.2}$$

where

$$S_{\text{Higgs}} = \int d^5x \sqrt{-g} \left[(D^A \Phi)^\dagger (D_A \Phi) \right] - \frac{1}{k} \int d^5x \sqrt{-g} \times \left\{ m^2 k + \left[\mu_H^2 + \frac{\lambda_H}{k} \Phi^\dagger \Phi \right] \delta(y - \pi r_c) - \left[\mu_P^2 - \frac{\lambda_P}{k} \Phi^\dagger \Phi \right] \delta(y) \right\} \Phi^\dagger \Phi, \tag{2.3}$$

and S_{gauge} is the usual action for the 5-d SM gauge fields. Note that in addition to the bulk mass m^2 for the Higgs field Φ we have allowed for mass terms on both the TeV and Planck branes: $\mu_{H,P}^2$. The bulk quartic term for the Higgs is a higher dimension operator, thus presumably suppressed, and is not demanded by gauge invariance in our 5-d effective theory. We thus set it to zero in the bulk. However, this term is marginal on the 4-d boundary theory and thus present with coefficients $\lambda_{H,P}$ on the TeV and Planck branes, respectively. Since we will be interested in first generating a tachyonic Higgs mass and then shifting the field as in the SM, the role of the quartic terms is just to stabilize the potential allowing for a positive mass-squared physical 4-d Higgs.

In order to generate EWSB, the Higgs action above must lead to (at least one) vev in 4-d. In the absence of brane terms one may try to shift the field Φ and then perform a KK decomposition as is usually done. This approach will not work here for several reasons. First, we note that the absence of a 5-d potential which could lead to a shift of the Higgs field by a constant amount. In order to construct a Higgs potential we first must KK decompose the Higgs field which, together with the quartic terms, will result in an effective 4-d potential with the desired properties once y is integrated over. This then allows us to shift the (as we will see) tachyonic ‘would-be’ zero mode by a constant amount identifying the resulting physical field with the Higgs. However, even when brane terms are absent, a scalar field with a bulk mass in a warped geometry does not possess a flat mode [14]. In addition, it has been observed that the mass eigenvalue of the first mode moves exponentially quickly from zero to order one as the bulk mass is turned on [4, 14], so we expect a vev with non-trivial bulk profile.

Our procedure will be as follows: we first consider the case of the free, non-interacting Higgs action and solve the corresponding bulk equations of motion with the brane mass terms, $\mu_{P,H}^2$, supplying the appropriate boundary conditions. Using the free Higgs action allows us to perform the KK decomposition. In certain regions of the parameter space this leads to a single light tachyonic scalar mode which we can identify as the unshifted Higgs field. Next we examine the full potential for this tachyonic mode and perform the usual shift in the field making direct connection with the SM.

The truncated Higgs action (i.e., ignoring gauge and self-couplings as well as other particles such as the Goldstone bosons so that we take $\Phi \rightarrow \phi$) S_{trunc} is given by

$$S_{\text{trunc}} = \int d^5x \sqrt{-g} \left[(\partial^A \phi)^\dagger (\partial_A \phi) - m^2 \phi^\dagger \phi + \frac{1}{k} \phi^\dagger \phi [\mu_P^2 \delta(y) - \mu_H^2 \delta(y - \pi r_c)] \right]. \quad (2.4)$$

To scale out dimensional factors and to make contact with our later development we define $m^2 = 20k^2\xi$ and $\mu_{P,H}^2 = 16k^2\xi\beta_{P,H}$ since k is the canonical scale for RS masses. Note that the parameters $\xi, \beta_{P,H}$ are dimensionless and are expected to be $O(1)$ but may, in principle, be of arbitrary sign. The choice of these unusual looking factors will be made clear below.

What are we looking for in the $(\xi, \beta_{P,H})$ parameter space? To obtain EWSB in a manner consistent with the SM our basic criterion is to find regions where there exists one, and only one, TeV scale tachyonic mode that we can identify with the SM Higgs. The remaining Higgs KK tower fields must also be normal, i.e., non-tachyonic. Clearly, if *none*

of the three mass terms in the action above are tachyonic no light tachyon will occur in the free Higgs spectrum.

To go further, we must obtain the relevant expressions for the Higgs KK masses and wavefunctions so we let $\phi \rightarrow \sum_n \phi_n(x)\chi_n(y)$. Substituting this expression into the action above and following the usual KK decomposition procedure leads to the equation of motion for the Higgs KK wavefunctions, χ_n :

$$\partial_y \left(e^{-4\sigma} \partial_y \chi_n \right) - m^2 e^{-4\sigma} \chi_n + \frac{1}{k} e^{-4\sigma} [\mu_P^2 \delta(y) - \mu_H^2 \delta(y - \pi r_c)] \chi_n + m_n^2 e^{-2\sigma} \chi_n = 0, \quad (2.5)$$

with m_n being the Higgs KK mass eigenvalues. The solutions are of the familiar form [14]

$$\chi_n = \frac{e^{2\sigma}}{N_n} \zeta_\nu(x_n \epsilon e^\sigma), \quad (2.6)$$

with $N_n \propto k^{-1/2}$ a normalization factor, $m_n = x_n k \epsilon$, $\epsilon = e^{-\pi k r_c} \simeq 10^{-16}$, $\nu^2 = 4 + m^2/k^2 = 4 + 20\xi$, and $\zeta_\nu = J_\nu + \kappa_n Y_\nu$, being the usual Bessel functions. While the κ_n are determined by the boundary conditions on the Planck brane and are generally very small, of order $\epsilon^{2\nu}$, the values of the x_n are determined from the boundary condition on the TeV brane. Explicitly one finds

$$-\kappa_n = \frac{\left[2 \left(1 + \frac{\mu_P^2}{4k^2} \right) - \nu \right] J_\nu(x_n \epsilon) + x_n \epsilon J_{\nu-1}(x_n \epsilon)}{\left[2 \left(1 + \frac{\mu_P^2}{4k^2} \right) - \nu \right] Y_\nu(x_n \epsilon) + x_n \epsilon Y_{\nu-1}(x_n \epsilon)}, \quad (2.7)$$

while the x_n roots can be obtained from

$$\left[2 \left(1 + \frac{\mu_H^2}{4k^2} \right) - \nu \right] \zeta_\nu(x_n) + x_n \zeta_{\nu-1}(x_n) = 0. \quad (2.8)$$

Since the κ_n are quite small the values of the parameters β_P and, correspondingly, μ_P^2 , are generally numerically irrelevant to the analysis that we will perform below (even for the case of $\nu = 0$ provided β_P is $O(1)$). Given the remaining two parameters, there are four possible sign choices to consider and we need to explore the solutions of the equations above in all these cases. However, we find that if the combination $\xi\beta_H > 0$ ($\mu_H^2 > 0$) then either no tachyon exists or that the resulting Higgs vev is Planck scale, independent of the sign of m^2 . This corresponds to an x_n root of the equation above whose value is of order ϵ^{-1} ; recall that we are seeking a single tachyonic root of order unity since $m_n = x_n k \epsilon$ and we expect that $k\epsilon$ to be at most of order a few hundred GeV in order to solve the hierarchy. Clearly, we must instead choose the parameters such that $\mu_H^2, \xi\beta_H < 0$. In this case the two remaining regions,

- (I) $\xi > 0, \beta_H < 0$
- (II) $\xi < 0, \beta_H > 0$

are found to be quite distinct since in the former case $\nu^2 > 0$ is guaranteed and thus ν is always real.

What conditions are necessary in these two regions in order to find only one, single $O(1)$ tachyonic root, x_T ? Our first goal is to find *at least* one such root which is $O(1)$. Let us assume that ν is real (and positive without loss of generality). Analytically, the best way to find a tachyonic root which is $O(1)$ is to look for the conditions on the parameters necessary to obtain a zero-mode and then to perturb around these. The root equation, eq. (8), above already tells us that if the term in the square brackets is zero then $x_n \zeta_{\nu-1}(x_n) = 0$, implying a root $x_n = 0$; thus if $\nu = 2 + 8\xi\beta_H = 0$ we obtain a zero mode. Similarly, if we set the term in the square bracket equal to -2ν and use the Bessel function identity, $2\nu\zeta_\nu(z) = z[\zeta_{\nu+1}(z) - \zeta_{\nu-1}(z)]$, we obtain $x_n \zeta_{\nu+1}(x_n) = 0$, which again has a zero root, and implies $-\nu = 2 + 8\xi\beta_H = 0$. Since $\nu \geq 0$, by hypothesis, these two equations lead directly to a pair of bounds on ξ which are necessary to satisfy in order to obtain $O(1)$ tachyonic roots: we first obtain the bound $\xi \geq (\leq)\xi_1$ in region I(II). Recalling, in addition, that $\nu^2 = 4 + 20\xi = (2 + 8\xi\beta_H)^2 \geq 0$, yields a second constraint $\xi \geq (\leq)\xi_2$ in region I(II). The $\xi_{1,2}$ are given by the expressions

$$\begin{aligned} \xi_1 &= -\frac{1}{4\beta_H} \\ \xi_2 &= \frac{5 - 8\beta_H}{16\beta_H^2}. \end{aligned} \tag{2.9}$$

Observe that in region I, ν is always real since $\xi \geq 0$ there. Furthermore, if these constraints are satisfied we find there exists only a single $O(1)$ tachyonic root, provided that ν is real. Interestingly, if these two constraints are not satisfied one finds no tachyons whatsoever so that EWSB cannot occur in those parameter space regions.

What do these constraints look like in regions I and II? Figure 1 shows all of the constraints in the $\xi - \beta_H$ plane for both regions I and II. Note that in region II, since $\xi < 0$, to maintain our assumption that ν is real requires that $\xi \geq -0.2$ as is also shown in the figure. In region I, since it is always true that $\xi_2 > \xi_1$, only the requirement that $\xi > \xi_2$ as a function of β_H is relevant and the allowed region is rather simple. This is not the case in region II where $\xi_1 = \xi_2$ when $\beta_H = 5/4$. In fact with the assumption that ν is real we are forced to have $\beta_H \geq 5/4$ in region II since there are now three simultaneous constraints to satisfy. What if we give up this $\nu^2 > 0$ assumption which populates the dominant part of region II? When $\xi < -0.2$ then ν is purely imaginary and one finds a multitude of $O(1)$ and larger tachyonic roots, thus violating our requirements that only one such root should exist. In addition, because ν is imaginary the factor $e^{2\nu}$ in κ_n becomes a rapidly varying phase leading to extreme parameter sensitivity. This is apparently a pathological regime where it is not obvious that any consistent EWSB scenario can be constructed. Thus from now on we will ignore the possibility that ν may be imaginary.

Figure 2 shows the possible values of the tachyonic root, x_T , as a function of ξ for a wide range of β_H values in both allowed regions. We see that for a significant range of the ξ and β_H parameters that indeed x_T is of order unity. It is important to observe that the value $\beta_H = 1$ is outside of the region II allowed range. Further restrictions on the parameter space will occur when we consider the gauge boson masses in more detail in the next section.

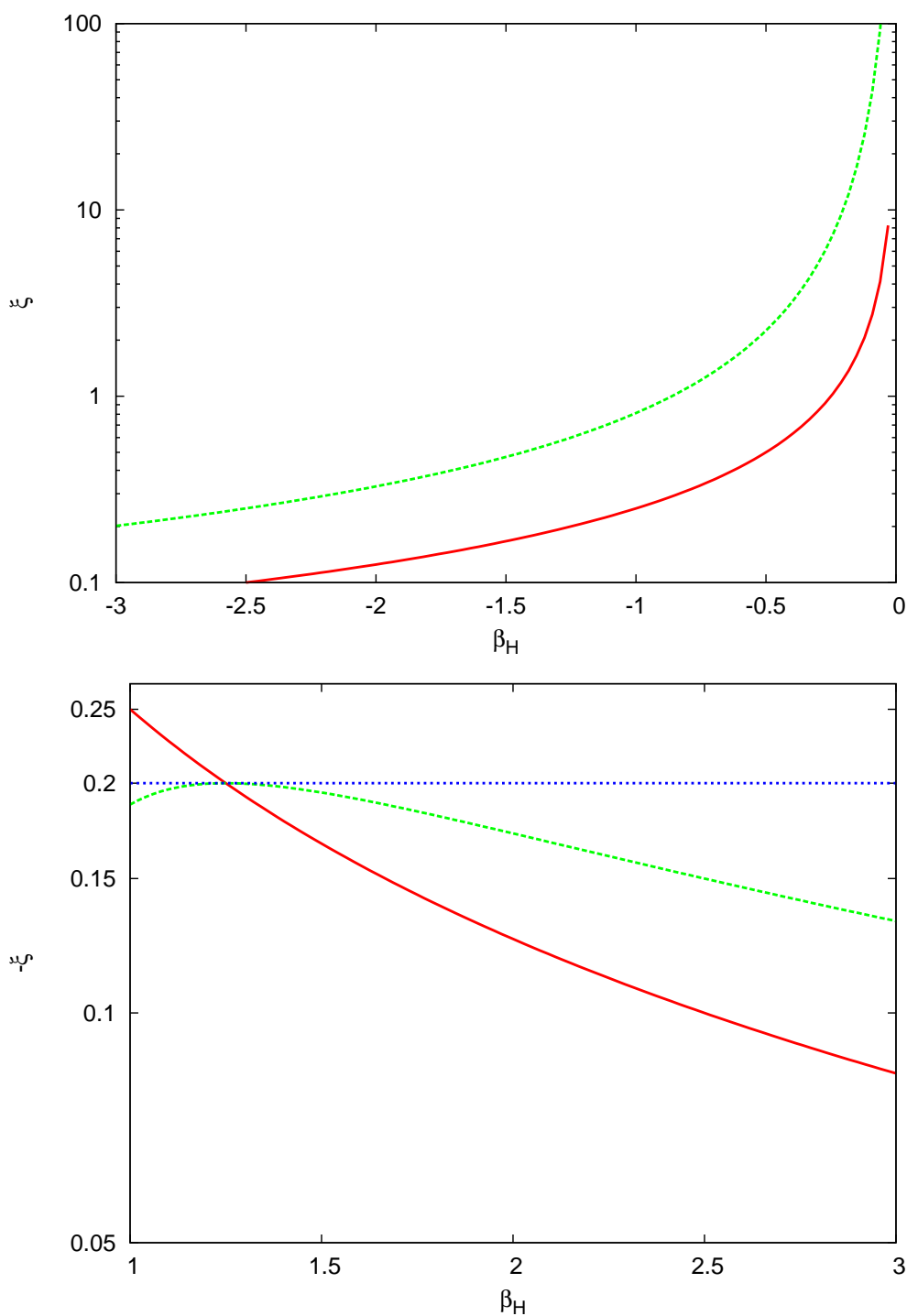


Figure 1: Allowed regions in the $\xi - \beta_H$ plane for bulk Higgs induced EWSB in region I (top) and region II (bottom). The lower bound $\xi \geq -0.2$ (dotted blue) that insures $\nu^2 \geq 0$ in region II is also shown in addition to both constraints ξ_1 (dashed green) and ξ_2 (in solid red). The allowed region lies between the blue and green curves in region II and above the green curve in region I.

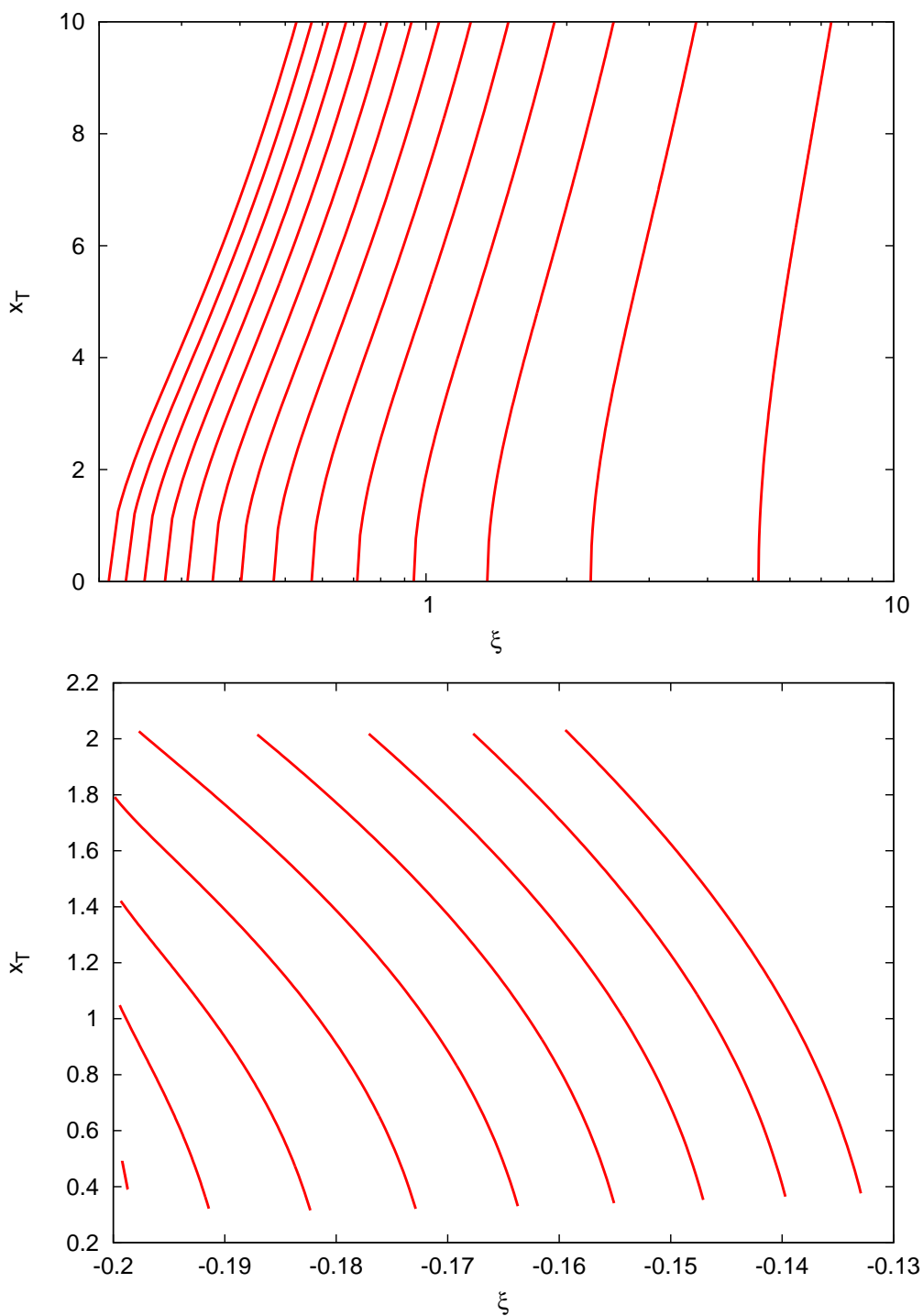


Figure 2: Tachyon roots as a function of ξ for different β_H . In region I (top), with β_H ranging from -0.3 to -2.9 , going from right to left in steps of 0.2 and with β_H values from 1.4 to 3.0 , going left to right, in steps of 0.2 for region II (bottom).

Noting the mass of the Higgs tachyonic field and inserting the Higgs KK decomposition $\phi = \sum_n \phi_n(x)\chi_n(y) = \phi_T\chi_T + \dots$, back into the action, integrating over y and extracting

only the resulting potential for the 4-d part of the tachyonic field, ϕ_T , yields the familiar expression

$$V = -m_T^2 \phi_T^\dagger \phi_T + \lambda_4 (\phi_T^\dagger \phi_T)^2, \quad (2.10)$$

where

$$\lambda_4 = \frac{\lambda_H}{k^2} \int dy \sqrt{-g} \chi_T^4(y) \delta(y - \pi r_c), \quad (2.11)$$

so that we can shift the field $\phi_T \rightarrow (v+H)/\sqrt{2}$ as usual. Here, we note that the contribution from the Planck brane quartic term is exponentially suppressed and is ignored. There are several things to note about this: (i) the physical ‘SM’ Higgs mass is given by $m_H = \sqrt{2}m_T = \sqrt{2}x_T k \epsilon$ and is fixed relative to the masses of all the other KK states and (ii) although the tachyonic ϕ mass is on the TeV brane, and possibly in the bulk as well, the Higgs vev now has a non-trivial profile in the 5-d bulk given by $\chi_T/\sqrt{2}$. As we can easily see this function is very highly peaked near the TeV brane since $\chi_T \sim e^{(2+\nu)\sigma}$. This is why we ‘see’ a TeV scale vev and Higgs boson mass and also why we avoid some of the previous problems with placing the Higgs field in the RS bulk [4]. Note that the overall shape of the Higgs profile is *adjustable* by varying the parameters ξ and β_H . (iii) It is important to observe that the vev, v , in the expression above is of the same scale as that of the SM Higgs, ~ 246 GeV, and is *not* Planckian.

It is important to note that the presence of the quartic terms in the 4-d effective potential that leads to a positive mass squared for the Higgs produces a small shift in the Higgs wavefunction relative to the vev profile. In lowest order of perturbation theory we can write this shift as

$$\chi_H(y) = \chi_T(y) - \frac{3}{2} \sum_{n=1} \frac{x_T^2}{(x_T^2 + x_n^2)} R_{1n} \chi_n(y), \quad (2.12)$$

where R_{1n} is just the ratio of wavefunctions

$$R_{1n} = \chi_n(\pi r_c) / \chi_T(\pi r_c) \quad (2.13)$$

which is less than unity as we will be demonstrated below. Here x_n are the roots corresponding to the Higgs KK excitations. Since as we will later see, $(x_n/x_T)^2 \gtrsim 100$, the above correction is at the level of a per cent and thus will be neglected in the discussion that follows, i.e., $\chi_H = \chi_T$, will be assumed from now on.

As the addition of the quartic term in the potential leads to a shift in the mass of the physical Higgs making it non-tachyonic, other terms in the 4-d quartic potential, $\sim \phi_T^2 \phi_n^2$, lead to small modifications in the masses of the Higgs KK states when we shift the field $\phi_T \rightarrow (v+H)/\sqrt{2}$. A short calculation leads to the result

$$\Delta m_n^2 = \frac{3}{4} m_H^2 R_{1n}^2. \quad (2.14)$$

We will return to this point further below where we will see that this shift is at most on the per cent level since $R_{1n} < 1$ and the KK masses are large compared to m_H .

3. Gauge boson masses

The masses of the SM gauge bosons are, as usual, generated via the kinetic terms in S_{Higgs} . Unlike the usual RS-type scenario, the Higgs vev is no longer restricted to the TeV brane but has a profile, χ_T , in the extra dimension. To extract the gauge boson mass terms from S_{Higgs} we can perform the standard KK decomposition and combine these terms with those obtained from the relevant pieces of S_{gauge} obtaining the gauge boson wavefunctions and KK mass spectra. To be specific, let us consider the case of the W boson. Suppressing Lorentz indices, we employ the KK decomposition $W = \sum_n W_n(x)f_n(y)$ and obtain the following equation for the gauge KK states:

$$\partial_y \left(e^{-2\sigma} \partial_y f_n \right) - \frac{1}{4} g_5^2 v^2 \chi_T^2 e^{-2\sigma} f_n + m_n^2 f_n = 0, \quad (3.1)$$

where g_5 is the 5-d $SU(2)_L$ gauge coupling. It is important to notice that (i) the 4-d Higgs TeV-scale vev, v appears here, *not* a 5-d vev and (ii) the tachyonic Higgs profile is present in the mass generating term. Here, we have included the back-reaction of the bulk Higgs profile on the gauge field KK equation of motion. Neglecting this back-reaction would have yielded the “free” field wavefunctions for the gauge KK fields, starting with a massless mode. To proceed, one would then integrate over the bulk degrees of freedom, using the free wavefunctions, and obtain a 4-d mass matrix for the W KK modes. To perform a diagonalization, one would have to truncate this mass matrix, in practice. The off-diagonal elements of this matrix are not small compared to the mass squared of the lightest W mode, and hence we do not expect this procedure to yield accurate results, using a modest truncation. Since we are going to compare the properties of the lightest W mode with those required from precision electroweak data, we choose to consider the exact equation above instead.

Now we would like to solve this equation to obtain the W KK spectrum and wavefunctions; however, due to the presence of $\chi_T^2(y)$, which is a combination of Bessel functions of imaginary argument, an analytic solution is not obtainable. We may, however, obtain a fairly good approximate solution by remembering that χ_T is strongly peaked at the TeV brane in which case $\chi_T^2 \rightarrow \epsilon^{-2} \delta(y - \pi r_c)$. One can thus solve the resulting equation exactly, but then we would have no idea how good our approximation is. We will refer to this as the ‘ δ -function limit’. The overall validity of this approximation will certainly improve for larger ξ since then $\chi_T^2 \sim e^{2(\nu+2)\sigma}$ becomes even more sharply peaked near the TeV brane in this case.

To obtain a better approximation, we performed the following calculation: we let $\chi_T^2 \rightarrow \lambda \epsilon^{-2} \delta(y - \pi r_c)$, where λ here is a free parameter, and solved this equation analytically for the usual KK spectrum and eigenfunctions. We then treat the difference

$$V_{pt} = \frac{1}{4} g_5^2 v^2 \left[\chi_T^2 e^{-2\sigma} - \lambda \delta(y - \pi r_c) \right], \quad (3.2)$$

as a first order ‘perturbing potential’ and calculate the elements of the W mass squared matrix for the KK states. Lastly, we vary the parameter λ until this mass-squared matrix is as diagonal as possible. In particular, we are specifically interested in making the off-diagonal elements in the first row and column be as small as possible as these influence

the W mass via mixing. Our expectation is that λ will be near unity if our approximation is valid. Performing this analysis for several choices of the input parameters tells us that indeed $\lambda \simeq 1.1 - 1.2$ in region I where ν is large and, in region II, $\lambda \simeq 1.1 - 1.3$ where ν is smaller. Thus the δ -function limit is a reasonable approximation in both regions and can be improved upon by choosing a λ in the above range. We now have gotten the W KK masses and eigenfunctions to a very good approximation. Obtaining the KK masses etc. for the other SM gauge fields can be done in a parallel manner. To connect with the 4-d SM we must also relate the 5-d coupling, g_5 , to the usual SM g . If the SM fermions are localized near either brane this can be done by defining the weak coupling as that between the brane fermions and the would-be W zero mode, i.e., $g^2 = g_5^2 f_0(y=0, \pi r_c)^2$. If BKT's for the gauge fields are present, this definition is easily modified to include such effects [12].

From this analysis we can extract the ‘roots’ corresponding to the mass of, e.g., the W boson in the usual manner, i.e., $M_W = x_W k\epsilon$. We typically find that $x_W \simeq 0.20 - 0.25$ from which we may infer that $k\epsilon \sim 350$ GeV. Using this result we would conclude that $x_T = 1$ corresponds to a Higgs mass of ~ 500 GeV so that this suggests that somewhat smaller values of $x_T \sim 0.5$ might be considered more favorable. From figure 3 we see that it is easy to obtain such values in both regions I and II as long as we live near the $\xi_{1,2}$ boundaries. How are the W and Z masses correlated in this model? Without extending the gauge group in the bulk to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, as would be done in a more realistic model [6, 17], there is no custodial symmetry present to insure the validity of the SM $M_W = M_Z \cos \theta_w$ relationship, i.e., $\rho = 1$. As discussed in ref.[15], if $v/(k\epsilon) \lesssim 1$ then the gauge KK mass scales approximately linearly in v and we can obtain $\rho \simeq 1$. This relation will become more exact as $v/(k\epsilon)$ becomes smaller. However, when $v/(k\epsilon)$ becomes very large then the generated gauge boson mass becomes independent of v and we would thus obtain $M_W = M_Z$, a gross violation of custodial symmetry. Since we sit between the two extremes, $v \sim k\epsilon$, we certainly would expect such violations to be of some significance in the present case. Sampling the model parameter space we do indeed find sizable deviations of ρ from unity due to custodial symmetry violation. Here we make use of the approximate gauge boson mass matrix calculations described above, and obtain deviations which are of order 5–10%, some of which may arise from the approximations we have made in obtaining the roots and wavefunctions. These deviations can be cured through an extension of the SM gauge group which enforces custodial symmetry as discussed above.

4. Gravity-induced electroweak symmetry breaking

As an example of EWSB with a bulk Higgs we now turn to the special case where EWSB is triggered by gravity.

4.1 Gravity action

In this case we need to consider a generalized version of the usual RS gravitational action which augments the above action:

$$S = S_{\text{bulk}} + S_{\text{branes}} + S_{\text{Higgs}} + S_{\text{gauge}} , \tag{4.1}$$

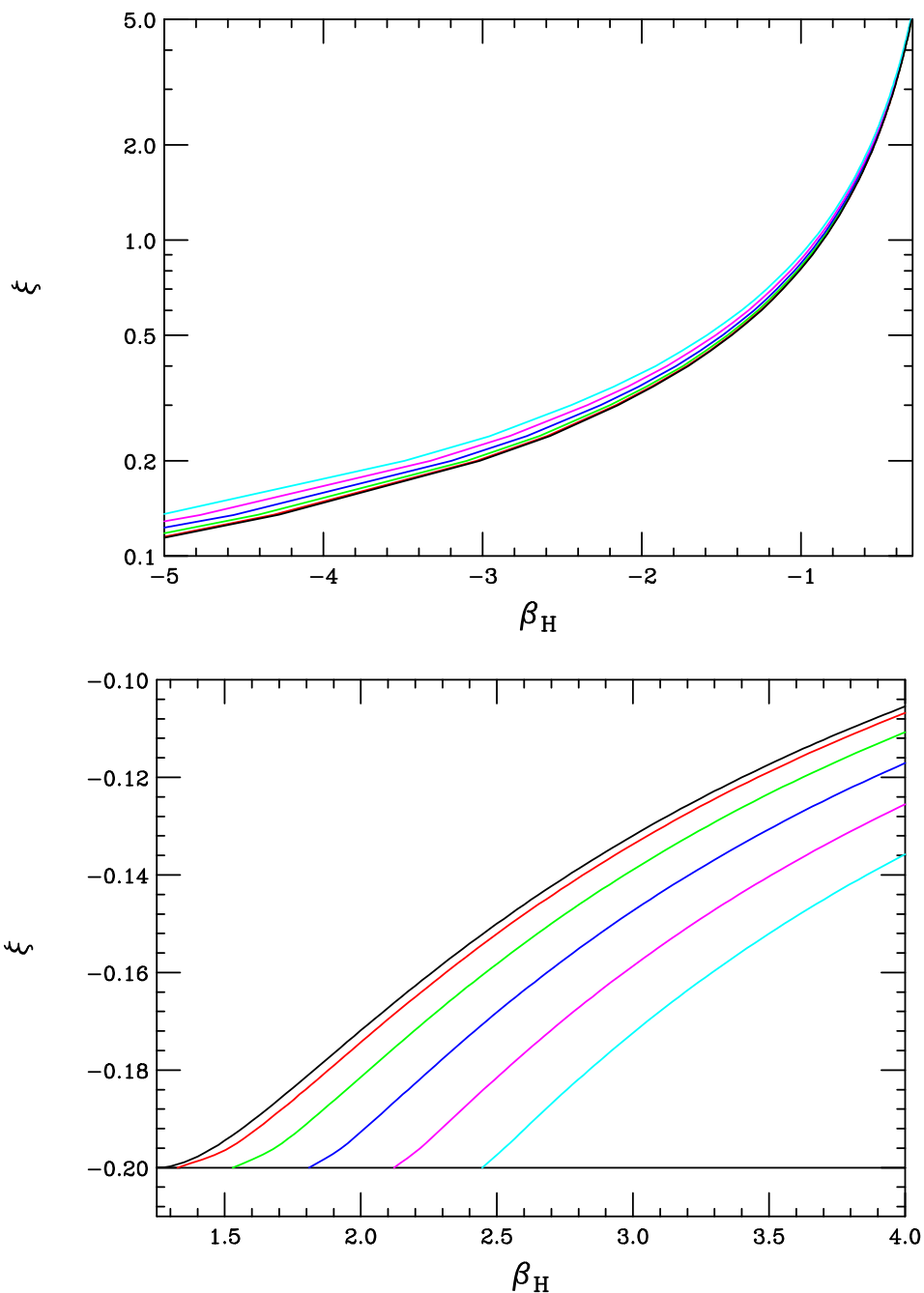


Figure 3: The curves correspond to fixed tachyon root values, $x_T = 0$ (black) to 2.5 (cyan), in steps of 0.5, as functions of ξ and β_H in regions I (top) and II (bottom).

where now the various terms are given by

$$S_{\text{bulk}} = \int d^5x \sqrt{-g} \left[\frac{M^3}{2} R - \Lambda_b + \frac{\alpha M}{2} \left(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \right) \right]$$

$$\begin{aligned}
 S_{\text{branes}} &= \sum_{\text{branes}} \int d^5x \sqrt{-g} \left[\frac{M^3}{k} \gamma_i R_4 - \Lambda_i - \frac{\lambda_i}{k^2} (\Phi^\dagger \Phi)^2 \right] \delta(y - y_i) \\
 S_{\text{Higgs}} &= \int d^5x \sqrt{-g} \left[(D^A \Phi)^\dagger (D_A \Phi) + \xi R \Phi^\dagger \Phi \right], \tag{4.2}
 \end{aligned}$$

and S_{gauge} is as above. In these expressions M is the 5-d Planck or fundamental scale, R is the usual 5-d Ricci curvature with Λ_b being the bulk cosmological constant, all of which are familiar from the usual RS model. Generally, k and M are expected not to be very different in magnitude in order to avoid introducing another hierarchy problem. The piece of S_{bulk} which is quadratic in the curvature is the GBT with α being a dimensionless parameter which we take to be of indefinite sign and roughly of $O(1)$. The γ_i are graviton BKT's for the TeV ($i = \pi$) and Planck ($i = 0$) branes and R_4 are the 4-d Ricci scalars derived from the corresponding induced metrics on both branes. The Λ_i are the usual RS brane tensions. The term $\xi R \Phi^\dagger \Phi$ in S_{Higgs} is the *bulk* Higgs-curvature mixing term with ξ soon to be identified with the parameter introduced in section 2. This is the most general form allowed for the gravitational action in the RS model framework.

There are no bare Higgs mass terms of any kind in this action as they are assumed to be generated from the Ricci scalar R . In order to generate the Higgs mass terms let us briefly recall that in the usual RS scenario, the 5-d Ricci scalar obtains a 'vev', i.e., the RS metric itself produces a background value for R . Employing the conventional RS relationships between the bulk and brane tensions, Einstein's equations lead to $\langle R \rangle = -20k^2 + 16k[\delta(y) - \delta(y - \pi r_c)]$ so that with, e.g., $\xi > 0$, $\langle R \rangle$ induces a positive mass squared for the Higgs in the bulk and on the TeV brane and a negative mass squared on the Planck brane. (Note that the original RS model requires that $\beta_H = \beta_P = 1$.) This would lead to a 4-d theory where the Higgs vev is of Planck scale as we found previously so that we must instead choose $\xi < 0$. However, in that case we saw that $\beta_H \geq 5/4$ is required to obtain EWSB. Thus, unless we alter the RS model in some way we cannot achieve gravity-induced EWSB.

The important role played by the GBT in this action is to maintain the general properties of the RS model while allowing an extension of the parameter space, i.e., to $\beta_H \neq 1$, for this scenario to be phenomenologically successful. As was recognized long ago by Kim, Kyae and Lee and subsequently discussed by other authors [11], the GBT allows us to modify the relationship between the tensions of the TeV and Planck branes and the other RS parameters which adds additional flexibility in the model. In particular one now finds that

$$\Lambda_{\text{Planck}} = -\Lambda_{\text{TeV}} = 6kM^3 \left(1 - \frac{4\alpha k^2}{3M^2} \right) \equiv 6kM^3 \beta_H, \tag{4.3}$$

so that the gravity induced effective free Higgs action is just

$$S_{\text{eff}} = \int d^5x \sqrt{-g} \left[(\partial^A \Phi)^\dagger (\partial_A \Phi) - m^2 (\Phi^\dagger \Phi) + \frac{\mu^2}{k} \Phi^\dagger \Phi [\delta(y) - \delta(y - \pi r_c)] \right], \tag{4.4}$$

and we identify $m^2 = 20k^2 \xi$ and $\mu^2 = 16k^2 \xi \beta_H$ as above in section 2. Given our previous general analysis, the allowed parameter space for gravity-induced EWSB is *already* known.

4.2 Classical stability

With the addition of gravity to our original action, the ξ term leads to a number of new effects, in particular mixing among the whole Higgs tower and radion fields. To address these issues requires several steps: First, we must extract from the action the kinetic term for the radion and see that it is properly normalized. Although this is straightforward it is non-trivial for the case at hand. Here we follow the work of Csáki, Graesser and Kribs (CGK) [8] and expand the metric as in their eq. (3.2):

$$ds^2 = e^{-2\sigma-2F} \eta_{\mu\nu} dx^\mu dx^\nu - [1 + 2F]^2 dy^2. \quad (4.5)$$

We write their quantity F as $e^{2\sigma} r_0(x)$. Next we insert this metric into the S_{bulk} , S_{branes} and S_{Higgs} terms in the action, perform a series expansion keeping terms only through second order in the derivatives of r_0 , and integrate over y dropping terms which are subleading in ϵ . This results in a kinetic term for r_0 of the form

$$\frac{6M^3}{k} (\partial r_0)^2 N_r^2, \quad (4.6)$$

where N_r^2 sums over several distinct contributions:

$$N_r^2 = \left(1 - 4\alpha \frac{k^2}{M^2}\right) (1 - 2\Omega_\pi), \quad (4.7)$$

with

$$\Omega_{0,\pi} \equiv \frac{4\alpha k^2/M^2 \pm \gamma_{0,\pi}}{1 - 4\alpha k^2/M^2}. \quad (4.8)$$

Note that N_r^2 contains the usual contributions from the Ricci scalar as well as those from both the GBT and BKT's. This quantity must be positive definite to avoid ghosts in the radion sector and this can be much more easily accomplished in region II where β_H is positive and α is negative.

As has been discussed by several authors [11], the presence of the BKT's and GBT in the RS model can result in the graviton and/or radion field becoming a ghost and the possible presence of a physical tachyon in the graviton spectrum; avoiding these problems constrains the model parameters as we saw in the case of N_r^2 above. Furthermore, eliminating the possibility of tachyons in the gravity sector requires [11] that the parameter $\Omega_\pi < 0$. Given this condition $N_r^2 > 0$ is automatically satisfied in region II while seemingly very difficult to satisfy in region I.

To make contact with TeV scale physics we must relate M^3/k to \overline{M}_{Pl}^2 ; recall that in the simple RS scenario $M^3/k = \overline{M}_{Pl}^2$ neglecting terms of order ϵ^2 . It turns out that in the case of the graviton, the requirement that the norm of this field also be positive definite, i.e., be ghost-free, is the same as requiring that \overline{M}_{Pl}^2 be positive definite when expressed in terms of the parameters in the action. Writing

$$\overline{M}_{Pl}^2 = \frac{M^3}{k} N_g^2, \quad (4.9)$$

a straightforward calculation[16] leads to the relation

$$N_g^2 = \left(1 - 4\alpha \frac{k^2}{M^2}\right)(1 + 2\Omega_0) + \frac{\xi k v^2}{M^3 \epsilon^2} > 0, \tag{4.10}$$

where again $O(\epsilon^2)$ terms have been neglected and we have taken the δ -function approximation for simplicity in the last term since it is ~ 0.01 or less. It is difficult for us to obtain $N_{r,g}^2 > 0$ in region I simultaneously. In region II, to obtain $N_g^2 > 0$ we only require that $\Omega_0 > -1/2$ which is a rather mild constraint. Note that if both $N_{r,g}^2 > 0$ then the graviton KK's also have positive definite norms. A curious observation is that there exists a small but finite region of the parameter space where no graviton brane terms are required to obtain both $N_{r,g}^2 > 0$; in such a case $\Omega_0 = \Omega_\pi \gtrsim -1/2$. Finally, putting all these pieces together we can write the normalized radion field, r , as

$$r_0 = r \frac{\epsilon^2}{\sqrt{6}\Lambda_\pi} \frac{N_g}{N_r}, \tag{4.11}$$

where $\Lambda_\pi = \overline{M}_{Pl}\epsilon$. Observe that in the original RS model both $N_{r,g} = 1$; the ratio $N = N_g/N_r$ will occur frequently in the expressions below.

Thus we conclude that we can obtain a working model of gravity-induced EWSB provided we live in region II. As shown in figures 1 and 3, typical values of ξ in region II are close to the conformal value $\xi = -3/16$.

4.3 Higgs-radion mixing

Since the normalized form of the radion field is now known, we can return to the action and extract out pieces which are quadratic in the scalars, perform the associated KK decompositions (ignoring for now the KK expansions of the Goldstone bosons which do not mix with the radion as we will see below) and integrate over y leaving us with the 4-d effective Lagrangian (a sum over n is implied)

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}H_n \square H_n - \frac{1}{2}m_{H_n}^2 H_n^2 - \frac{1}{2}m_r^2 r^2 \\ & + \xi \gamma A_n H_n \square r - \frac{1}{2} \left[1 + B \xi \gamma^2 \right] r \square r, \end{aligned} \tag{4.12}$$

which is analogous to that obtained by CGK [8] in their eq. (10.12); here $\gamma = \frac{v}{\sqrt{6}\Lambda_\pi} \simeq 0.01 - 0.05$. Defining $n = T, i$ with ‘ T ’ labeling the mode that gets a vev, the coefficients are given by

$$\begin{aligned} A_T &= 2\epsilon^2 N \int dy \chi_T^2 \simeq 2N \\ A_i &= 2\epsilon^2 N \int dy \chi_T \chi_i \simeq 0 \\ B &= -6\epsilon^4 N^2 \int dy e^{2k|y|} \chi_T^2 \simeq -6N^2. \end{aligned} \tag{4.13}$$

where the approximate results hold in the δ -function limit. Employing a scan of the parameter space we find that these approximations hold to better than $\simeq 50\%$; the true

values tend to be somewhat below those given by the δ -function approximation, e.g., $A_T \simeq 1.3(1.7)$ and $-B \simeq 3.4 - 4.1(4.8 - 5.5)$ in region II(I). Here we make some note of the factor ‘2’ appearing in the definition of A_n above; this value is a result of the mixing terms being bulk operators with the full 5-d Ricci scalar. In the standard case of wall Higgs fields, $R \rightarrow R_{\text{ind}}$ and ‘2’ \rightarrow ‘6’ and the results of CGK are recovered. The kinetic mixing can be removed as usual by suitable field redefinitions:

$$\begin{aligned} H_n &\rightarrow H'_n + \xi\gamma\beta A_n r' \\ r &\rightarrow \beta r', \end{aligned} \tag{4.14}$$

with

$$\beta^{-2} = 1 + B\xi\gamma^2 - \xi^2\gamma^2 A_n^2, \tag{4.15}$$

where a sum on the index $n = (T, i)$ is now understood. Demanding that β^{-2} be positive places another constraint on our model parameters, in particular, we obtain a bound on the parameter ξ :

$$\frac{B}{2A_n^2} \left[1 + \left(1 + \frac{4A_n^2}{\gamma^2 B^2} \right)^{1/2} \right] \leq \xi \leq \frac{B}{2A_n^2} \left[1 - \left(1 + \frac{4A_n^2}{\gamma^2 B^2} \right)^{1/2} \right]. \tag{4.16}$$

Given our parameter space this bound is easily satisfied. Mass mixing remains but this can be easily read off from \mathcal{L} above and follows the usual course [8].

5. Experimental signatures

In this section we will discuss some of the phenomenological features of our model as well as how our framework may be experimentally tested.

In a fully realistic model, all SM fields should live in the bulk. The 5-d fermion masses will be chosen so that the overlap of the would-be zero mode with χ_T produces the correct Yukawa couplings. Some mechanism also needs to be introduced to protect the ρ parameter, such as enlarging the gauge group to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [6, 17]. It is known that, in this type of model with a Higgs on the IR brane, there is a tension between producing the correct top mass and not causing too large a shift in the $Zb\bar{b}$ coupling. With the non-trivial profile χ_T this problem should be softened, since the left-handed top (and hence the left handed bottom) field can be moved closer to the Planck brane.

5.1 Higgs KK excitations

Our framework describes a wide class of potential models with a new feature: the fundamental SM Higgs field is in the RS bulk and thus has a scalar KK excitation spectrum [18]. This is a completely new feature previously not considered within the RS model structure. Once the values of β_H and ξ are specified in our model so are the ratios of the masses of the Higgs KK’s to that of the usual Higgs. (We will ignore radion mixing in this discussion for simplicity since it is likely to have little influence on the Higgs KK excitations.) Exploring this parameter space one generically finds that the first Higgs KK excitation has a mass

$\sim 30 - 100(15 - 30)$ times larger than the W in region I(II), with the largest ratios obtained for ξ near its boundary value. This suggests a mass range for the lightest Higgs KK state of $\gtrsim 1 - 1.5$ TeV and is generally found to be more massive than the first graviton KK excitation. To address this point, figure 4 shows the root values for the first Higgs KK as a function of β_H for different values of ξ in both regions. Here we see that almost all of the time the Higgs KK is more massive than the first graviton KK. It is important to remember that the masses of all the Higgs KK excitations are fixed once the values of ξ and β_H are known.

We remind the reader that the physical masses of the Higgs KK excitations are slightly shifted from the values given by the root equation, $x_n k \epsilon$, due to the presence of the quartic term in the potential. Given the expression for Δm_n^2 above we obtain

$$m_n^2 = \left[x_n^2 + \frac{3}{2} x_T^2 R_{1n}^2 \right] (k\epsilon)^2. \tag{5.1}$$

Since R_{1n} are found to be < 1 and for almost all parameter space regions of interest $x_T/x_n \leq 0.1$ it is clear that the magnitudes of these shifts in the Higgs KK spectrum are at most at the per cent level and can be neglected in our discussion below.

Can such a KK state be observed at planned colliders? Unless the couplings are very large the Higgs KK states seem to be beyond the range of the LHC but may be produced by Z bremsstrahlung at multi-TeV linear colliders such as CLIC. To address this issue we must ask how a Higgs KK couples to SM fields. To obtain the $H_n WW$ -type couplings we return to the Higgs kinetic term in S_{Higgs} and extract out the term which is bilinear in the W field yet linear in the Higgs. Then we perform the usual KK decomposition for each field and extract the relevant coupling which we can express as an integral over y :

$$S_{\text{eff}} = \int d^4x \sum_n H_n W_\mu^\dagger W^\mu \frac{1}{2} g_5^2 v \int dy e^{-2\sigma} \chi_T \chi_n f_0^2. \tag{5.2}$$

Interestingly we see that if the W wave-function were flat the integral would vanish by orthogonality suggesting a small coupling. However, we also know that that the most significant deviation in flatness for this gauge boson wavefunction occurs near the TeV brane where χ_T is largest. To set the scale for this coupling and to make the connection with the conventional SM WWH coupling strength, $g_{WWH}^{SM} = g^2 v/2$, we note that if we take the light fermions to be localized near the Planck brane, we can define $g_5^2 = 2\pi r_c g^2$. We can then perform the integral above and scale the resulting WWH_1 effective coupling to the SM value. A scan of the parameter space in both regions I and II indicates that the coupling of H_1 to WW is substantially suppressed in comparison to the SM WWH value. We find that $g_{WWH_1}/g_{WWH}^{SM} \simeq 0.02 - 0.13$ with the larger values appearing in region II. With these rather small couplings it is clear that a large integrated luminosity will be necessary to detect the H_1 KK state if it is produced by gauge boson fusion or radiated off a gauge boson leg.

It is interesting to note that a similar expression to the above with $\chi_n \rightarrow \chi_T$ allows us to calculate the usual Higgs coupling to the W . Ordinarily one would imagine that this is purely proportional to M_W^2 as in the SM. Here, due to the back-reaction of the gauge boson

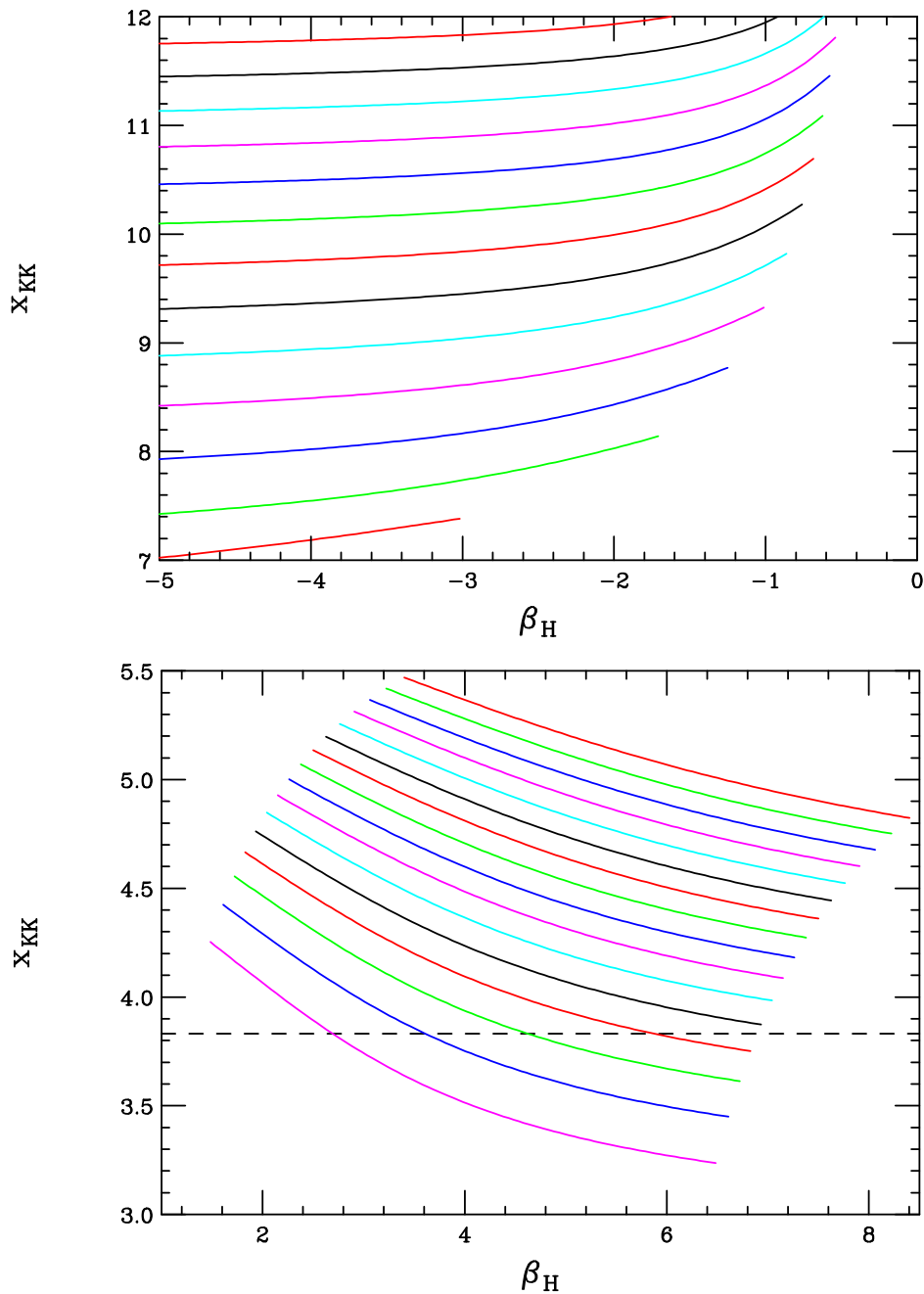


Figure 4: Roots for the first Higgs KK excitation as a function of β_H for different values of ξ in regions I (top) and II (bottom). In region I, $\xi = 0.2$ is the lowest curve with ξ increasing by 0.2 for each subsequent curve. The curves are cut off on the right hand side by the ξ constraints. In region II, from top to bottom ξ runs from -0.120 to -0.195 in steps of 0.005. The largest possible value for the first graviton KK root consistent with the constraint $\Omega_\pi \leq 0$ found in the case of gravity induced breaking is shown as the dashed line.

wavefunction this is no longer the case. This can be seen from eq. (3.1) by setting $n = 0$, multiplying by f_0 , integrating over y and solving for $m_0^2 = M_W^2$. Using orthonormality,

integrating by-parts and employing the usual $\partial_y f_0 = 0$ boundary conditions on both branes gives

$$M_W^2 = \frac{1}{4} g_5^2 v^2 \int dy e^{-2\sigma} f_0^2 \chi_T^2 + \int dy e^{-2\sigma} (\partial_y f_0)^2, \quad (5.3)$$

where the first term on the right is directly due to the Higgs boson vev while the second corresponds to back-reaction. Here we can see explicitly that these two components are of the same sign; from this we can conclude that the HW coupling in this model is less than in the SM. This shift in the coupling of the Higgs to gauge bosons can be significant and can be measured at the LHC/ILC. As in the case of H_1 above, we can scale the WWH effective coupling by the corresponding SM value. A numerical scan of the parameter space in regions I and II indicates that $g_{WWH}/g_{WWH}^{SM} \simeq 0.45 - 0.70$, with the larger values obtained in region II. This large shift in coupling strength will be easily observable at the ILC and likely also to be seen at the LHC. Of course the appearance of a suppression of the WWH coupling in the RS model framework is not limited to the present scenario and is thus not a unique feature of the present scheme[17].

The Higgs self-interactions as well as the Higgs KK's coupling to SM Higgs can be obtained through the quartic self-couplings appearing in S_{Higgs} . To this end, we remind ourselves that $\Phi^\dagger \Phi$ can be written as

$$\Phi^\dagger \Phi = \frac{1}{2} (v + H)^2 \chi_T^2 + (v + H) \chi_T \sum_n \chi_n H_n + \frac{1}{2} \sum_{n,m} \chi_n \chi_m H_n H_m + G^+ G^- + \frac{1}{2} G^0 G^0, \quad (5.4)$$

where here the G 's are the non-KK expanded 5-d Goldstone particles. We first observe that there are no trilinear or quartic couplings between a *single* Goldstone KK and the SM Higgs; however, such terms for the couplings of two Higgs to a single Higgs KK excitation do exist. To evaluate these terms we first observe that we can immediately relate λ_H to the SM quartic coupling, $\lambda_{SM} = m_H^2/(2v^2)$, via the integral

$$\lambda_{SM} = \frac{\lambda_H}{k^2} \int dy \sqrt{-g} \chi_T^4 \delta(y - \pi r_c). \quad (5.5)$$

Thus the $H_n HH$ and $H_n HHH$ couplings can be obtained from the action

$$S'_{\text{eff}} = \sum_n \lambda_{eff_n} \int d^4x (3vH^2 + H^3) H_n, \quad (5.6)$$

where

$$\lambda_{eff_n} = \lambda_{SM} R_{1n}. \quad (5.7)$$

In a similar manner, one can calculate the equivalent of the SM Higgs trilinear coupling; one finds that one recovers the SM expression when the definition of $\lambda_{5\Phi}$ in terms of λ_{SM} given above is employed. A sampling of the model parameter space indicates that $\lambda_{eff_1}/\lambda_{SM} \simeq 0.65 - 0.91$ so that the $H_1 \rightarrow HH$ decay mode branching fraction will be sizable.

Given the properties of the H_1 KK state it is likely that this particle can be most easily produced in gg -fusion or in $\gamma\gamma$ collisions which can proceed through top quark loops. The reason for this is that with the top in the RS bulk it is likely that the $H_1 t\bar{t}$ coupling will remain reasonably strong. Another possibility would thus be the associated production of an H_1 together with $t\bar{t}$.

5.2 Direct tests

In the case of gravity-induced EWSB there are strong correlations between the Higgs and gravity sectors. Although there are many parameters in the model, they are already constrained by the analysis above and they are all likely to be accessible through future collider experiment. This implies that the scenario becomes overconstrained and the model structure can be directly tested. As we will see, such measurements will require LHC/ILC and likely a multi-TeV e^+e^- collider for detailed studies of both the scalar and graviton sector. The reason for this is clear: we need precision measurements and the masses of some of the important states can be in excess of 1 TeV.

Let us begin with the gravity sector: We assume that the first three graviton KK excitations, $G_{i=1,2,3}$, are accessible and that their properties can be determined in detail at e^+e^- colliders. A measurement of the *relative* KK masses gives us Ω_π as this mass ratio depends only on this single parameter and any individual mass then provides the quantity $k\epsilon$. A measurement of the ratio of partial widths for the same final state, e.g., $\Gamma(G_2 \rightarrow \mu^+\mu^-)/\Gamma(G_1 \rightarrow \mu^+\mu^-)$, yields a determination of a combination of the parameters $\Omega_{0,\pi}$ while an *overall* width measurement yields us Λ_π . The graviton KK spectrum is such that $G_3 \rightarrow 2G_1$ is kinematically allowed [19] and the possibility of studying such decays in detail at e^+e^- colliders has already been discussed in the literature [20]. The rate and angular distribution for this processes is sensitive to both the existence of BKT's as well as the GBT [10] allowing us to separate these two contributions once Λ_π is known. However, such precision measurements of G_3 are likely to require a multi-TeV linear collider. When $\Omega_\pi \lesssim -0.28$, the decay $G_2 \rightarrow 2G_1$ is also allowed with different contributing weights arising from the BKT's and GBT contributions. In either case, it is likely that the graviton sector alone will tell us α , $\gamma_{0,\pi}$, Λ_π , k/M and hence β_H . As one can see, a combination of just these measurements is already rather restrictive and may be sufficient to confirm or exclude the present model.

Now for the scalar sector: We assume that the two lightest states, which are mixtures of radion with the Higgs, can both be observed so that their masses and couplings can be well determined. From these data it will be possible to reconstruct the mixing matrix which provides for us the 'weak' eigenstate mass parameters as well as a determinations of ξ . Given ξ , the Higgs and first KK Higgs masses, the value of β_H can again be extracted and compared with that obtained from the gravity sector. A confirmation of the model is obtained if the two values agree.

6. Conclusions

The consistency of having a fundamental 5-d Higgs doublet in the RS geometry was studied in this work. We showed that by assigning appropriate bulk and brane masses for the 5-d "Off-the-Wall" Higgs, one can achieve a realistic 4-d picture of EWSB, without fine-tuning. In our construct, the SM Higgs is the lightest KK mode of the bulk doublet which in the 4-d reduction has a tachyonic mass, leading to EWSB. Since all of the SM fields are now in the RS bulk, this scenario represents an example of a *warped* Universal Extra Dimension.

Previous attempts at EWSB with a bulk Higgs field in the RS geometry were plagued by extreme fine-tuning and phenomenological problems [4]. Nearly all such models had considered endowing the bulk Higgs with a 5-d constant vev which yielded massive SM gauge bosons in 5 dimensions. However, in our framework, EWSB involves only one KK mode of the bulk Higgs and is hence localized. The resulting Higgs vev has a bulk-profile that is nearly identical to that of the physical SM Higgs. Although the consistency of the bulk Higgs scenario imposes non-trivial constraints on our framework, we observe that a realistic 4-d phenomenology can be achieved for a range of parameters.

A particularly interesting realization of the Off-the-Wall Higgs EWSB employs the gravitational sector of the RS model to provide the necessary bulk and brane mass scales. These scales are then related to the 5-d curvature of the RS geometry. Consequently, new relations and constraints among the parameters of the Higgs and gravitational sectors are obtained. In this “gravity-induced” EWSB scenario, the higher curvature Gauss-Bonnet terms play an important role. The coupling of the gravity and Higgs sectors as well as the introduction of the higher derivative terms affect the classical stability of the RS geometry. We find that one of the two regions of parameter space is favored by these stability considerations. As we have shown, typical values of ξ in region II are close to the conformal value $\xi = -3/16$.

We outlined the experimental tests and phenomenological features of the Off-the-Wall Higgs and gravity-induced EWSB in our work. A novel feature of our general bulk scenario is the existence of an adjustable 5-d profile of the Higgs vev which can provide a new tool for model building. Generically, we also expect the appearance of TeV scale Higgs KK modes. In the case of gravity-induced EWSB, observation of radion-Higgs mixing, together with measurements of the Higgs and graviton KK modes can provide tests of this mechanism at future colliders; TeV-scale e^+e^- colliders are well-suited for this purpose.

In summary, we have presented a consistent framework for placing the Higgs doublet in the RS geometry bulk and achieving EWSB at low energies. The possibility that 5-d gravity drives 4-d EWSB is considered and shown to be a viable option. With the Higgs residing in the RS bulk, the entire SM can be thought of as a 5-d theory and it also becomes feasible to think of bulk gravity as the cause of EWSB. This provides a cohesive 5-d picture of all the known forces of nature that is both predictive and free of large hierarchies.

Note added. After this paper was essentially completed, ref. [21] was brought to our attention where the possibility of using the bulk curvature in the RS model to generate the Higgs vev was also considered. As discussed in this work, the parameter space point employed by these authors leads to multiple tachyons in the scalar spectrum and is plagued by extreme parameter sensitivity.

Acknowledgments

H. Davoudiasl would like to thank K. Agashe, D. Chung and F. Petriello for discussions related to this paper. This work was supported in part by the U.S. Department of Energy under grant DE-FG02-95ER40896 and contract DE-AC02-76SF0051. The research of H.D.

is also supported in part by the P.A.M. Dirac Fellowship, awarded by the Department of Physics at the University of Wisconsin-Madison. The authors would also like to thank the Aspen Center of Physics for their hospitality while this work was being completed.

References

- [1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [2] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phenomenology of the Randall-Sundrum gauge hierarchy model*, *Phys. Rev. Lett.* **84** (2000) 2080 [[hep-ph/9909255](#)].
- [3] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Bulk gauge fields in the Randall-Sundrum model*, *Phys. Lett.* **B 473** (2000) 43 [[hep-ph/9911262](#)]; *Experimental probes of localized gravity: on and off the wall*, *Phys. Rev.* **D 63** (2001) 075004 [[hep-ph/0006041](#)];
A. Pomarol, *Gauge bosons in a five-dimensional theory with localized gravity*, *Phys. Lett.* **B 486** (2000) 153 [[hep-ph/9911294](#)];
Y. Grossman and M. Neubert, *Neutrino masses and mixings in non-factorizable geometry*, *Phys. Lett.* **B 474** (2000) 361 [[hep-ph/9912408](#)];
T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [[hep-ph/0003129](#)].
- [4] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, *Bulk standard model in the Randall-Sundrum background*, *Phys. Rev.* **D 62** (2000) 084025 [[hep-ph/9912498](#)];
S.J. Huber and Q. Shafi, *Higgs mechanism and bulk gauge boson masses in the Randall-Sundrum model*, *Phys. Rev.* **D 63** (2001) 045010 [[hep-ph/0005286](#)]; see also the second paper in ref. [3].
- [5] LEP collaboration, *A combination of preliminary electroweak measurements and constraints on the standard model*, [hep-ex/0412015](#).
- [6] C. Csáki, C. Grojean, H. Murayama, L. Pilo and J. Terning, *Gauge theories on an interval: unitarity without a Higgs*, *Phys. Rev.* **D 69** (2004) 055006 [[hep-ph/0305237](#)];
C. Csáki, C. Grojean, L. Pilo and J. Terning, *Towards a realistic model of higgsless electroweak symmetry breaking*, *Phys. Rev. Lett.* **92** (2004) 101802 [[hep-ph/0308038](#)];
G. Cacciapaglia, C. Csáki, C. Grojean and J. Terning, *Oblique corrections from higgsless models in warped space*, *Phys. Rev.* **D 70** (2004) 075014 [[hep-ph/0401160](#)]; *Curing the ills of higgsless models: the S parameter and unitarity*, *Phys. Rev.* **D 71** (2005) 035015 [[hep-ph/0409126](#)];
G. Cacciapaglia, C. Csáki, C. Grojean, M. Reece and J. Terning, *Top and bottom: a brane of their own*, *Phys. Rev.* **D 72** (2005) 095018 [[hep-ph/0505001](#)];
C. Csáki, C. Grojean, J. Hubisz, Y. Shirman and J. Terning, *Fermions on an interval: quark and lepton masses without a Higgs*, *Phys. Rev.* **D 70** (2004) 015012 [[hep-ph/0310355](#)];
Y. Nomura, *Higgsless theory of electroweak symmetry breaking from warped space*, *JHEP* **11** (2003) 050 [[hep-ph/0309189](#)];
R. Barbieri, A. Pomarol and R. Rattazzi, *Weakly coupled higgsless theories and precision electroweak tests*, *Phys. Lett.* **B 591** (2004) 141 [[hep-ph/0310285](#)];
H. Davoudiasl, J.L. Hewett, B. Lillie and T.G. Rizzo, *Higgsless electroweak symmetry breaking in warped backgrounds: constraints and signatures*, *Phys. Rev.* **D 70** (2004) 015006 [[hep-ph/0312193](#)]; *Warped higgsless models with IR-brane kinetic terms*, *JHEP* **05** (2004) 015 [[hep-ph/0403300](#)];

- J.L. Hewett, B. Lillie and T.G. Rizzo, *Monte Carlo exploration of warped higgsless models*, *JHEP* **10** (2004) 014 [[hep-ph/0407059](#)];
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. Tanabashi, *The structure of corrections to electroweak interactions in higgsless models*, *Phys. Rev. D* **70** (2004) 075008 [[hep-ph/0406077](#)]; *Universal non-oblique corrections in higgsless models and beyond*, *Phys. Lett. B* **603** (2004) 210 [[hep-ph/0408262](#)].
- [7] T. Appelquist, H.-C. Cheng and B.A. Dobrescu, *Bounds on universal extra dimensions*, *Phys. Rev. D* **64** (2001) 035002 [[hep-ph/0012100](#)];
- H.-C. Cheng, K.T. Matchev and M. Schmaltz, *Bosonic supersymmetry? getting fooled at the LHC*, *Phys. Rev. D* **66** (2002) 056006 [[hep-ph/0205314](#)]; *Radiative corrections to Kaluza-Klein masses*, *Phys. Rev. D* **66** (2002) 036005 [[hep-ph/0204342](#)];
- T.G. Rizzo, *Probes of universal extra dimensions at colliders*, *Phys. Rev. D* **64** (2001) 095010 [[hep-ph/0106336](#)].
- [8] G.F. Giudice, R. Rattazzi and J.D. Wells, *Graviscalars from higher-dimensional metrics and curvature- Higgs mixing*, *Nucl. Phys. B* **595** (2001) 250 [[hep-ph/0002178](#)];
- C. Csáki, M.L. Graesser and G.D. Kribs, *Radion dynamics and electroweak physics*, *Phys. Rev. D* **63** (2001) 065002 [[hep-th/0008151](#)];
- D. Dominici, B. Grzadkowski, J.F. Gunion and M. Toharia, *Higgs-boson interactions within the Randall-Sundrum model*, *Acta Phys. Polon. B* **33** (2002) 2507 [[hep-ph/0206197](#)]; *The scalar sector of the Randall-Sundrum model*, *Nucl. Phys. B* **671** (2003) 243 [[hep-ph/0206192](#)];
- J.L. Hewett and T.G. Rizzo, *Shifts in the properties of the Higgs boson from radion mixing*, *JHEP* **08** (2003) 028 [[hep-ph/0202155](#)].
- [9] See, however, K. Farakos and P. Pasipoularides, *Gravity-induced instability and gauge field localization*, *Phys. Lett. B* **621** (2005) 224 [[hep-th/0504014](#)].
- [10] B. Zwiebach, *Curvature squared terms and string theories*, *Phys. Lett. B* **156** (1985) 315;
- B. Zumino, *Gravity theories in more than four-dimensions*, *Phys. Rept.* **137** (1986) 109.
- [11] J.E. Kim, B. Kyae and H.M. Lee, *Various modified solutions of the Randall-Sundrum model with the Gauss-Bonnet interaction*, *Nucl. Phys. B* **582** (2000) 296 [[hep-th/0004005](#)],
erratum *ibid.* **591** (2000) 587;
- J.E. Kim, B. Kyae and H.M. Lee, *Effective Gauss-Bonnet interaction in Randall-Sundrum compactification*, *Phys. Rev. D* **62** (2000) 045013 [[hep-ph/9912344](#)];
- K.A. Meissner and M. Olechowski, *Brane localization of gravity in higher derivative theory*, *Phys. Rev. D* **65** (2002) 064017 [[hep-th/0106203](#)]; *Domain walls without cosmological constant in higher order gravity*, *Phys. Rev. Lett.* **86** (2001) 3708 [[hep-th/0009122](#)];
- Y.M. Cho, I.P. Neupane and P.S. Wesson, *No ghost state of Gauss-Bonnet interaction in warped background*, *Nucl. Phys. B* **621** (2002) 388 [[hep-th/0104227](#)];
- I.P. Neupane, *Localized gravity with higher curvature terms*, *Class. and Quant. Grav.* **19** (2002) 5507 [[hep-th/0106100](#)];
- C. Charmousis and J.-F. Dufaux, *Gauss-Bonnet gravity renders negative tension braneworlds unstable*, *Phys. Rev. D* **70** (2004) 106002 [[hep-th/0311267](#)];
- Y. Shtanov and A. Viznyuk, *Linearized gravity on the Randall-Sundrum two-brane background with curvature terms in the action for the branes*, *Class. and Quant. Grav.* **22** (2005) 987 [[hep-th/0312261](#)];
- P. Brax, N. Chatillon and D.A. Steer, *Effective actions of a Gauss-Bonnet brane world with brane curvature terms*, *Phys. Lett. B* **608** (2005) 130 [[hep-th/0411058](#)];

- T.G. Rizzo, *Warped phenomenology of higher-derivative gravity*, *JHEP* **01** (2005) 028 [[hep-ph/0412087](#)].
- [12] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Brane localized kinetic terms in the Randall-Sundrum model*, *Phys. Rev.* **D 68** (2003) 045002 [[hep-ph/0212279](#)];
M. Carena, E. Ponton, T.M.P. Tait and C.E.M. Wagner, *Opaque branes in warped backgrounds*, *Phys. Rev.* **D 67** (2003) 096006 [[hep-ph/0212307](#)];
F. del Aguila, M. Perez-Victoria and J. Santiago, *Physics of brane kinetic terms*, *Acta Phys. Polon.* **B34** (2003) 5511 [[hep-ph/0310353](#)]; *Bulk fields with brane terms*, *Eur. Phys. J.* **C33** (2004) S773 [[hep-ph/0310352](#)]; *Some consequences of brane kinetic terms for bulk fermions*, [hep-ph/0305119](#); *Bulk fields with general brane kinetic terms*, *JHEP* **02** (2003) 051 [[hep-th/0302023](#)].
- [13] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Brane localized curvature for warped gravitons*, *JHEP* **08** (2003) 034 [[hep-ph/0305086](#)];
S.L. Dubovsky and M.V. Libanov, *On brane-induced gravity in warped backgrounds*, *JHEP* **11** (2003) 038 [[hep-th/0309131](#)];
G. Kofinas, R. Maartens and E. Papantonopoulos, *Brane cosmology with curvature corrections*, *JHEP* **10** (2003) 066 [[hep-th/0307138](#)].
- [14] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [[hep-ph/9907447](#)].
- [15] For a discussion, see the second paper in ref. [12].
- [16] For an outline of the calculation and original references, see the last paper in ref. [11]
- [17] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [[hep-ph/0308036](#)];
B. Lillie, *Collider phenomenology of Higgs bosons in left-right symmetric Randall-Sundrum models*, *JHEP* **02** (2006) 019 [[hep-ph/0505074](#)].
- [18] See, however, A. Kehagias and K. Tamvakis, *Bulk Higgs with 4d gauge interactions*, *Phys. Lett.* **B 628** (2005) 262 [[hep-th/0507130](#)];
C.-H.V. Chang and J.N. Ng, *Bulk Higgs boson decays in brane localized gravity*, *Phys. Lett.* **B 488** (2000) 390 [[hep-ph/0006164](#)];
K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys.* **B 719** (2005) 165 [[hep-ph/0412089](#)];
G. Denardo and E. Spallucci, *Improved effective potential in the Einstein metric*, *Class. and Quant. Grav.* **6** (1989) 1915;
R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson*, *Nucl. Phys.* **B 671** (2003) 148 [[hep-ph/0306259](#)].
- [19] H. Davoudiasl and T.G. Rizzo, *Bulk physics at a graviton factory*, *Phys. Lett.* **B 512** (2001) 100 [[hep-ph/0104199](#)].
- [20] M. Battaglia, A. De Roeck and T. Rizzo, *Graviton production at clic*, *eConf* **C010630** (2001) E3022 [[hep-ph/0112169](#)].
- [21] A. Flachi and D.J. Toms, *Higgs mechanism in the Randall-Sundrum model*, *Phys. Lett.* **B 491** (2000) 157 [[hep-th/0007025](#)].